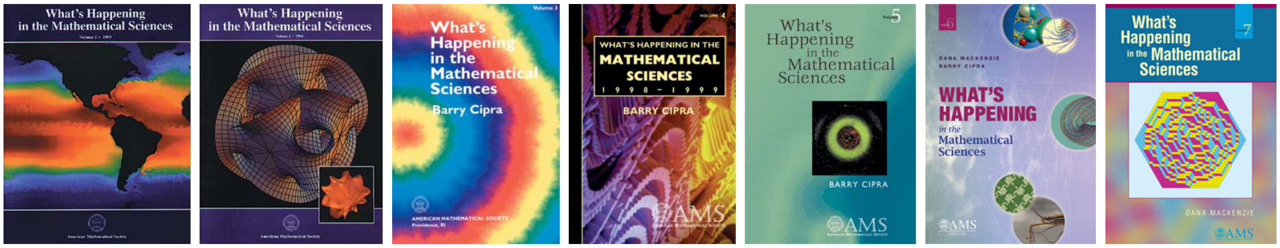
What's happening in the mathematical sciences, vol. 7 *Dana Mackenzie*, American Mathematical Society, 2009 (127 p.), soft cover, ISBN 978-0-821a-4478-6, US$15.95.



Covers of volumes 1-7 of *What's happening in the mathematical sciences*

What you see above are the covers of the first seven volumes of the AMS series *What's happening in the mathematical sciences*. I have here in front of me the latest volume 7 from 2009. The first five volumes appeared respectively in 1993, 1994, 1996, 1999, and 2002. They were all authored by Barry Cipra. Volume six is from 2006 and Dana Mackenzie entered the picture because Barry Cipra could not finish that one. With the latest volume the responsibility has completely been transferred to Dana Mackenzie.

Barry Cipra

The purpose of the series is to give on a regularly basis short introductions for a broad mathematical audience to recent evolutions in mathematics. There are around 10 topics per issue, each one taking about 10 pages, which are usually richly illustrated. Excerpts from some of the volumes can be found as google books on the web.

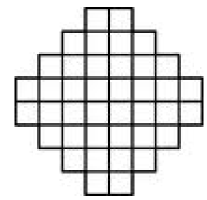
To come to volume seven, there are 9 contributions. They are all showcases that appeal to any reader with a general mathematical interest. There is not to much mathematics, in the sense that formulas are almost completely absent. But on the other hand it takes a minimal mathematical skill to understand what is actually going on. Indeed, a formula pops up once in a while, and then of course one should be able to understand series, matrices, summations, products, equations, coordinate systems, groups, etc. Nothing fancy, yet not for a mathematical illiterate either. Although not explicit, the chapters are written like one would write an interview with the mathematical experts for the problem being discussed. It is clear the Dana Mackenzie has talked to the researchers and the text is a sedimentation of these interviews, not a summary of published papers.

It is not a coincidence that new results emerge where previously unrelated areas meet. That is for example the case in the first contribution: *A new twist in knot theory*. The triggering result is that it was proved in 2006s by E. Ghys that a modular knot is topologically identical to a Lorenz knot. The first one living in number theory and the second one in dynamical systems. The article sketches the history and possible consequences of the result, even linking it with the Riemann Hypothesis (discussed in volumes 4 and 5 of this series). It is worthwhile to have a look at the beautifully illustrated online text by E. Ghys *Lorenz and modular flows: a visual introduction* at *http://www.ams.org/featurecolumn/archive/lorenz.html*, one of the AMS featured columns. These columns have a purpose very close to the purpose of the series under review.

Dana Mackenzie

The second article *Error-term roulette and the Sato-Tate conjecture* is again about number theory (clearly deterministic) but now interfering with probability theory (obviously stochastic). For example the Prime Number Theorem (1898) which roughly says that the probability of a number being prime is inversely proportional to the number of digits in the number. But such statements only hold 'on the average'. This links up again with the Riemann Hypothesis. The Sato-Tate conjecture is also about counting: how much deviates the number of points on an elliptic curve from the mean value, and what is the distribution of these deviations. That problem was solved in 2006. That proof is the result of an interaction of three major machineries: L-functions, automorphic forms and Galois representation theory, following a road-map pointed out by Jean-Pierre Serre. The article is mainly explaining this link between the three pillars.

The fifty-one percent solution is about the surprising observation that when tossing a coin, tossed vigorously enough and caught in midair has about a 51% chance of landing with the same face up as started with. This article describes the analysis of two Stanford researchers, Persi Diaconis and Susan Holmes, and Richard Montgomery from UC at Santa Cruz, analysing the dynamics of the normal vector on the coin when tossed. Their finding were published in 2007 in *Siam Review* Volume 49, Issue 2, pp. 211-235, that confirmed the observation.


Aztec diamond

Given a chessboard with two diagonally opposite corner squares removed, can you cover the remaining part with 31 dominos. The answer is no, and there is an easy proof. This is the prototype of the problems about random tilings considered in *Dominos, anyone*. If there is indeed a feasible solution, how many different ones do there exist, and how about a hexagon tiled with lozenges (see cover page of this volume) or rhombi; or tiling an Aztec diamond? Investigating the random tilings of Aztec diamonds of fixed size with dominos of decreasing size gives connections with statistical physics describing crystalline of liquid phases.

Not seeing is believing brings us at the verge of Harry Potter's invisibility cloak or Star Trek cloaking devises. In fact a first prototype of such a cloak was built in 2006 at Duke university. In fact invisibility for certain wavelengths in tomography, this is a serious problem. This is a contribution where physics dominates with e.g. Maxwell equations and magnetic fields, yet there is some interesting mathematics underneath.

The minimal model program by S. Mori (Fields Medal 1990) is an active research topic in algebraic geometry. It looks for the "simplest" birational model, i.e., a version of any complex variety that still has the same function space defined on it. *Getting with the (Mori) program* illustrates the historical background from the "Italian School" to the progress made in extending results from dimension 2 to higher dimensions.

In *The cook that time couldn't erase* the remarkable story is told about a palimpsest, i.e., a parchment book of which the pages had been scraped, turned over 90 degrees and written over. The original text was an account of writings by Archimedes, which had been transformed into a prayer book by monks in 1229. When it was sold during an auction in 1998, it was in a terrible state. However science as carefully recovered the original text (pet name "Archie") which is now publically available at *www.archimedespalimpsest.org* with the explanation of the recovery project and the tools used. This is an "Indiana Jones story" from real life.

The story in *Charting a 248-dimensional world* is about the Lie group E_8, with its 248 parameters, it is the largest of the exceptional groups in Killing's classification. Writing it as a combination of its irreducible representations requires a matrix that uses 60 gigabytes of data, 60 times the amount of data in the human gnome. Thanks to ingenious programming, the character map of E_8 was completed in 2007.

The last contribution is again applied. The title *Compressed sensing makes every pixel count* says indeed what the main idea is behind the buzz-word "compressed sensing". Almost everyone is making digital pictures now and it is also familiar that these big matrices of pixels (the number of mega pixels is considered to be a measure of quality of the camera) are then compressed to e.g., a jpeg format so that it tikes much less space to store. The purpose of compressed sensing is to reduce the number of pixels that are sensed, and then mathematically recover a good image from even an undersampled observation that does cover the information content of the image. For example, with only few characteristics, it is possible to reconstruct a human face. The "single-pixel camera" captures only few randomly chosen pixels to reconstruct the image. Hence mathematicians and engineers start to think beyond Shannon's sampling theorem.

Books, articles and websites like this one are multiplying fast and it is a fortunate sign that mathematicians take the trouble of explaining their work to a more general public, hopefully attracting young enthusiastic students and, why not, convince people of the importance of funding their work.

Adhemar Bultheel